



IB MYP YEAR 5
ASSESSMENT TASK
A Broad-based Test

SOLUTIONS

| | | | |
|----------------------------|---|--------------------------|-----|
| Subject: | Y10 <i>Standard</i> Mathematics | Name : (Class) | () |
| Assessment: | Broad-based Test | | |
| Topics covered | Polynomials, Transformations (and transforming functions), Probability, Vectors, Matrices, Indices | | |
| Date of assessment: | Thursday 16th February 2012 | | |

- This task assesses Criteria A and C;
- Time allowed – *one hour 40 minutes*;
- You must answer all the questions;
- Write your answers in the spaces provided;
- Show all of your working – not just the answer
- GDCs are allowed.

| Criterion A | | |
|-------------|--|---|
| Levels | Task-Specific Rubric | Official IB Descriptors |
| 0 | The student does not reach a standard described by any of the descriptors given below. | |
| 1-2 | Students are reasonably successful with the Part A questions only. Any errors here are relatively minor. | The student generally makes appropriate deductions when solving simple problems in familiar contexts. |
| 3-4 | Students are successful with Part A questions. The only errors in part B questions are minor. | The student generally makes appropriate deductions when solving more complex problems in familiar contexts. |
| 5-6 | Students are successful with Part A and B questions. The only errors in part C questions are minor. | The student generally makes appropriate deductions when solving challenging problems in a variety of familiar contexts. |
| 7-8 | Students are successful with Part A, B and C questions. The only errors in part D questions are minor. | The student consistently makes appropriate deductions when solving challenging problems in a variety of contexts including unfamiliar situations. |

| Criterion C | | |
|-------------|---|--|
| Levels | Task-Specific Rubric | Official IB Descriptors |
| 0 | The student does not reach a standard described by any of the descriptors given below. | |
| 1-2 | Very little working is shown, and/or the steps shown are confusing. Only the most basic mathematical symbols are used with accuracy. | The student shows basic use of mathematical language and/or forms of mathematical representation. The lines of reasoning are difficult to follow . |
| 3-4 | The working shown is generally adequate. Only a few errors in symbols/terminology are evident. It is reasonably easy to follow a student's logic/reasoning. | The student shows sufficient use of mathematical language and forms of mathematical representation. The lines of reasoning are clear though not always logical or complete . The student moves between different forms of representation with some success . |
| 5-6 | There are very few, if any, errors in symbols/terminology. All steps in calculations are shown in their completeness. It is easy to follow all the student's logic/reasoning. | The student shows good use of mathematical language and forms of mathematical representation. The lines of reasoning are concise, logical and complete . The student moves effectively between different forms of representation. |

Part A (Level 1-2 Questions)

Q1. Factorise **completely** the following expressions:

(a) $x^2 - 16$

$$= x^2 - 4^2$$

difference of 2 squares

Answer (a) $(x-4)(x+4)$

(b) $x^2 - 16x$

$$= x(x-16)$$

Answer (b) $x(x-16)$

(c) $x^2 - 16x - 36$

$$= (x-18)(x+2)$$

Answer (c) $(x-18)(x+2)$

Q2. Simplify the following expressions, giving your answers in the form a^n , where a and n are integers:

(a) $3^5 \times 3^{-4}$

$$= 3^{5+(-4)}$$

Answer (a) 3^1

(b) $2^4 \times 3^4$

$$= (2 \times 3)^4$$

Answer (b) 6^4

(c) $(6^2)^3$

$$= 6^{2 \times 3}$$

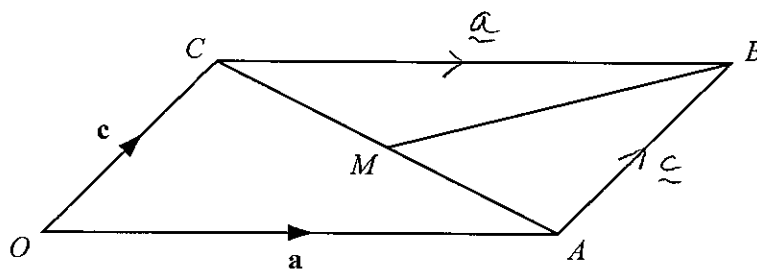
Answer (c) 6^6

(d) $16^2 \div 2^8$

$$= \frac{(2^4)^2}{2^8} = \frac{2^8}{2^8}$$

Answer (d) 2^0

Q3.



OACB is a parallelogram. $\vec{OA} = \underline{a}$, $\vec{OC} = \underline{c}$ and M is the midpoint of CA. Find, in terms of \underline{a} and \underline{c} :

(a) \vec{OB}

$$\begin{aligned}\vec{OB} &= \vec{OA} + \vec{AB} \\ &= \underline{a} + \underline{c}\end{aligned}$$

Answer (a) $\underline{a} + \underline{c}$

(b) \vec{CA}

$$\begin{aligned}\vec{CA} &= \vec{CO} + \vec{OA} \\ &= -\underline{c} + \underline{a}\end{aligned}$$

Answer (b) $\underline{a} - \underline{c}$

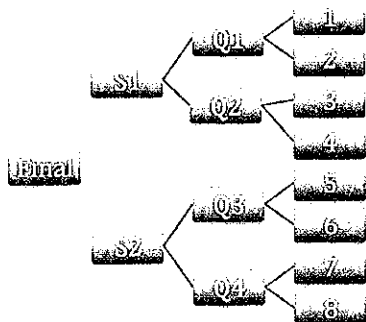
(c) \vec{BM}

$$\begin{aligned}\vec{BM} &= \frac{1}{2} \vec{BO} \quad (\text{diagonals of a parallelogram bisect each other}) \\ &= -\frac{1}{2} \vec{OB}\end{aligned}$$

Answer (c) $-\frac{1}{2}(\underline{a} + \underline{c})$

Part B (Level 3-4 Questions)

Q4. Eight teams take part in a basketball tournament. Each team is equally likely to win any particular game. Winning teams advance to the next stage. The losers go home!!



* This is a misleading question - all 8 teams are already in the Q-Finals !!

(a) Find the probability that all the even numbered team reach the quarter-finals (Q1, Q2, Q3, Q4). accept as "win"

$$p = (2 \text{ beats } 1 \text{ AND } 4 \text{ beats } 3 \text{ AND } 6 \text{ beats } 5 \text{ AND } 8 \text{ beats } 7)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} \quad (\text{independent events})$$

NB (see above*)
Logically we have to accept an answer of 1

Answer (a) $\frac{1}{16}$

(b) Find the probability that team 1 will play team 8 in the final.

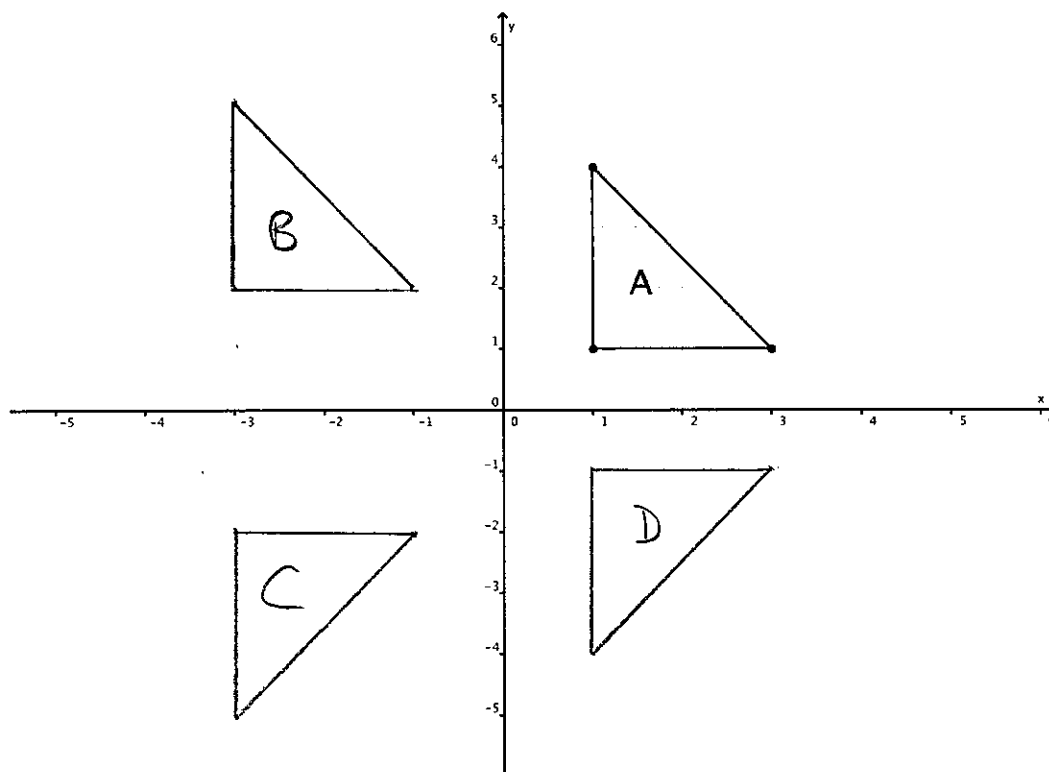
$$p(1 \text{ reaches final}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$p(8 \text{ " " "}) = \frac{1}{4} \text{ also}$$

$$\Rightarrow p(\text{both reach final}) = \frac{1}{4} \times \frac{1}{4} \quad (\text{independent events})$$

Answer (b) $\frac{1}{16}$

Q5. Triangle A is plotted on the Cartesian plane below.



- (a) On the same Cartesian plane above, draw the following:
- (i) Triangle A after it is translated by a translation vector $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$, label it B.
 - (ii) Triangle C is the reflection of triangle B in the x-axis, draw triangle C.
 - (iii) Triangle D is the reflection of triangle A in the x-axis, draw triangle D.
- (b) Describe the single transformation, which maps triangle C onto triangle D.

Translation 4 units right, 1 up

Answer $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Q6. Erik runs a race at an average speed of x m/s.

His time is $(3x - 9)$ seconds and the race distance is $(2x^2 - 8)$ metres.

(a) Write down an equation in x and show that it simplifies to

$$x^2 - 9x + 8 = 0$$

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

$$\Rightarrow x = \frac{2x^2 - 8}{3x - 9} \Rightarrow 3x^2 - 9x = 2x^2 - 8$$

$$\Rightarrow x^2 - 9x + 8 = 0 \quad \text{QED}$$

(b) Solve $x^2 - 9x + 8 = 0$

$$(x-8)(x-1) = 0$$
$$\Rightarrow x-8 = 0$$
$$\text{or } x-1 = 0$$

Answer (b) $x = 1$ or $x = 8$

(c) Write down Erik's time and the race distance.

We can ignore the $x=1$ solution, as it gives a negative value of time and distance

$$\Rightarrow x=8 \Rightarrow \text{time} = 3(8)-9 = 15 \text{ seconds}$$
$$\text{and distance} = 2(8)^2-8 = 120 \text{ m}$$

Answer (c) $15 \text{ s}, 120 \text{ m}$

Part C (Level 5-6 Questions)

Q7. Solve the following equation for x :

$$2^{x+3} + 2^x = 36$$

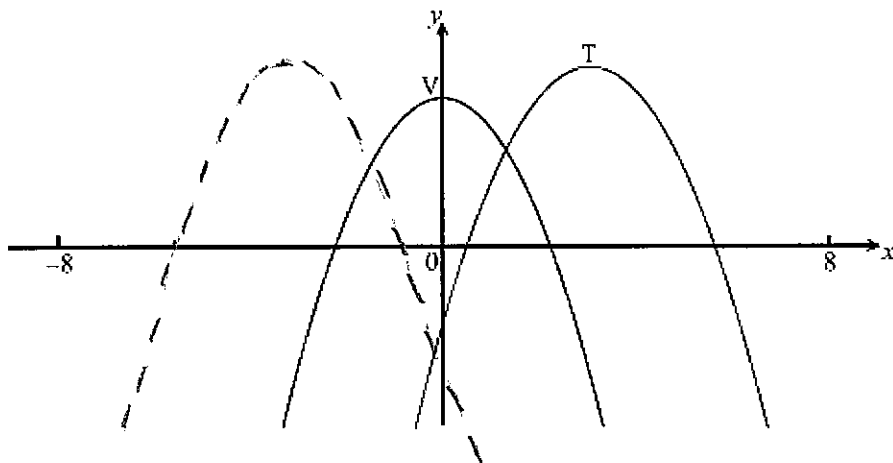
$$\text{Let } y = 2^x$$

$$\Rightarrow 8y + y = 36$$

$$\Rightarrow 9y = 36 \Rightarrow y = 4 \Rightarrow 2^x = 4$$
$$\Rightarrow x = 2$$

Answer $x = 2$

- Q8.** The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex $V(0, 5)$.
 Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex $T(3, 6)$.



- (a)** What are the value of h and k ?

$g(x)$ is formed by translating $f(x)$ by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

Answer (a) $h=3$ $k=1$

- (b)** Write down an expression for $g(x)$.

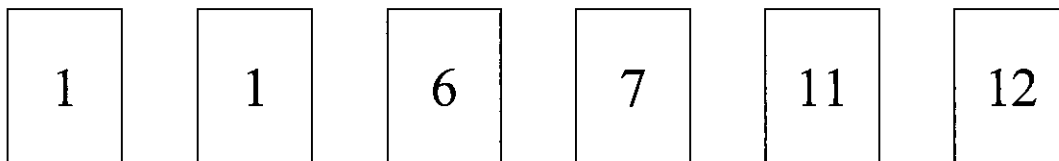
$$\begin{aligned} g(x) &= [5 - (x-3)^2] + 1 \\ &= 5 - (x^2 - 6x + 9) + 1 \\ &= 6x - x^2 - 3 \end{aligned}$$

Answer (b)

- (c)** On the same diagram, sketch the graph of $y = g(-x)$.

see sketch (dotted)
 (look for reflection of $g(x)$ in y -axis)

Q9.



Six cards are numbered 1, 1, 6, 7, 11 and 12 as shown.

Cards are chosen, without replacement, until a card numbered 1 is chosen.

Find the probability this happened **before** the fourth card is chosen.

$$\begin{aligned}
 p &= p(\text{1st Card} = 1) + p(\text{1st not 1, 2nd is}) + p(\text{1st, 2nd not 1, 3rd is}) \\
 &= \left(\frac{2}{6}\right) + \left(\frac{4}{6} \times \frac{2}{5}\right) + \left(\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4}\right) \\
 &= \frac{1}{3} + \frac{4}{15} + \frac{1}{5} \\
 &= \frac{5+4+3}{15} = \frac{12}{15}
 \end{aligned}$$

Answer $p = \frac{4}{5}$

Part D (Level 7-8 Questions)

Q10. Luis deposits a large sum of money in a bank account that pays 0.6% interest, compounded monthly. How long does it take Luis's money to grow by 10%?

After n months, Luis' money is worth $(1.006)^n X$
(where X is the principal)

We require: $(1.006)^n X = 1.1X$

$$\text{ie. } 1.006^n = 1.1$$

$$\Rightarrow n = 15.93$$

- May see trial+error
- May see use of logs
- May see graphical solution
- May see GDC "E-Solve" solution

NB if students thought the 0.6% was an annual rate, accept appropriate working and answer

Answer 16 months

- Q11.** A cubical die is biased, so that some faces are more likely to show than others.
The following table summarises the situation:

| Score | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------|-------|-------|-----|-------|------|------|
| Probability | p^2 | p^2 | p | $p/2$ | 0.03 | 0.05 |

- (a) Find the value of p

$$\begin{aligned}
 p^2 + p^2 + p + \frac{p}{2} + 0.03 + 0.05 &= 1 \\
 \Rightarrow 2p^2 + \frac{3p}{2} - 0.92 &= 0 \\
 \Rightarrow 4p^2 + 3p - 1.84 &= 0 \\
 \Rightarrow p = 0.4 &\quad (\text{ignore } p = -1.15 \text{ as it's impossible})
 \end{aligned}$$

Answer (a) $p = 0.4$

- (b) What is the most likely score when you roll this biased die?

| Score | 1 | 2 | 3 | 4 | 5 | 6 |
|-------|------|------|-----|-----|------|------|
| prob | 0.16 | 0.16 | 0.4 | 0.2 | 0.03 | 0.05 |

(by substituting $p = 0.4$ into table)

biggest probability = 0.4 (for a score of 3)

Answer (b) 3

- (c) If you roll the die twice and add the scores, what is the probability the total is at least 10?

$$\begin{aligned}
 P(\text{Score} \geq 10) &= P(\text{Score} = 10) + P(\text{Score} = 11) + P(\text{Score} = 12) \\
 &= P(4+6) + P(5+5) + P(6+4) + P(6+5) + P(5+6) + P(6+6) \\
 &= 2(0.2 \times 0.05) + (0.03)^2 + 2(0.03)(0.05) + (0.05)^2 \\
 &= 0.0264
 \end{aligned}$$

Answer (c) 0.0264